In the present lesson we consider the motion when the central acceleration follows Newtonian Law of Attraction.

This law may be expressed as follows between every two particles, of masses m_1 and m_2 placed at a distance r apart, the mutual attraction is

$$\gamma \frac{m_1 m_2}{r^2}$$

units of force, where γ is a constant, dependind on the units of mass and length employed, and known as the gravitation constant.

If the mass be measured in grammes and the length in centimetres, the value of γ is 6.66×10^{-8} approximately, and the ~~length~~ attraction is expressed in dynes.

If the masses be measured in kilogrammes and the length in met.

Now we consider a particle moves in a path so that its acceleration is always directed to a fixed point and is equal to $\frac{\mu}{(\text{distance})^2}$ to show that its path is a conic section and to distinguish between the three cases that arise.

$$P = \frac{h^2}{p^3}\frac{dp}{dr}$$

When $P = \frac{\mu}{r^2}$, the equation becomes

$$\frac{h^2}{p^3}\frac{dp}{dr} = \frac{\mu}{r^2} \quad \text{-------------- (1)}$$

Integrating $v^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$ we get

$$v^2 = \frac{h^2}{p^2}$$

$$= \frac{2\mu}{r} + c \quad \text{-------------- (2)}$$

Now the (p, r) equation of an ellipse and hyperbola, reffered to a focus are respectively.

$$\left.\begin{aligned} \frac{b^2}{p^2} &= \frac{2a}{r} - 1 \\ \text{and } \frac{b^2}{p^2} &= \frac{2a}{r} + 1 \end{aligned}\right\} \text{------------ (3)}$$

where $2a$ and $2b$ are the transverse and conjugate axes.

Hence when c is negative, (2) is an ellipse. When c is negative, it is a hyperbola.

Also when $c = 0$, (2) becomes $\frac{p^2}{r}$ = constant and this is the (p, r) equation of a parabola referred to its focus.

Hence (2) always represents a conic section, whose focus is at the centre of focus, and which is an

ellipse, parabola or hyperbola according as C is negative, zero or positive

i e according as $v^2 \lesseqgtr \frac{2\mu}{r}$, ie according as the square of the velocity at any point P is $\lesseqgtr \frac{2\mu}{SP}$ where S is the focus.

Again comparing eqns (2) and (3), we have in the case of ellipse,

$$\frac{h^2}{b^2} = \frac{\mu}{a} = \frac{C}{-1}$$

$$\therefore h = \sqrt{\mu \frac{b^2}{a}} = \sqrt{\mu \times \text{Semi-latus-rectum}}$$

and

$$C = -\frac{\mu}{a}$$

Hence, in the case of ellipse,

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \quad \text{------------------} (4)$$

So, for the hyperbola,

$$v^2 = \mu\left(\frac{2}{r} + \frac{1}{a}\right)$$

and, for the parabolla,

$$v^2 = \frac{2\mu}{r}$$

It will be noted that in each case the velocity at any point does not depend on the direction of the velocity.

Since h is twice the area described in the unit time. Therefore, if T be the time of describe the ellipse, we have

$$T = \frac{\text{area of the ellipse}}{\frac{1}{2}h}$$

$$= \frac{\pi ab}{\frac{1}{2}\sqrt{\mu \frac{b^2}{a}}}$$

$$= \frac{2\pi}{\sqrt{\mu}} a^2 \qquad \text{——————} (5)$$

So that the square of the periodic time varies as the cube of the major axis.

CoR. 1. If the particle is projected at a distance R with velocity V in any direction the path is an ellipse.

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